LNA Matching Techniques for Optimizing Noise Figures

It is common knowledge that amplifiers add noise and distortion to the desired signal. Various techniques can be employed to minimize such unwanted effects — LNA matching is a method that costs little, yet returns a lot.

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An RF amplifier is an active network that increases the amplitude of weak signals, thereby allowing further processing by the receiver. Receiver amplification is distributed between RF and IF stages throughout the system, and an ideal amplifier increases the desired signal amplitude without adding distortion or noise. Unfortunately, amplifiers add noise and distortion to the desired signal.

In a receiver chain, the first amplifier after the antenna contributes most to the system noise figure (assuming low loss in front of the amplifier). Adding gain in front of a noisy network reduces the noise contribution from that network.

The first part of this article is a refresher addressing the theoretical noise figure for a two-port RF network, and the optimum reflection coefficient for the minimum noise figure.

The rest deals with using scattering parameters (S parameters) as a design tool to match impedances for minimum noise figure. The analysis considers optimum noise matching for an SiGe low-noise amplifier.

Amplifier Noise Figure

Two methods are available for analyzing the effect of noise in electronic devices and circuits. The first method substitutes equivalent noise sources at appropriate physical locations in the small-signal model for the device. As an example, consider the noise produced by two resistors in series (figure 1a). The noise model of a resistor (figure 1b) produces an open-circuit voltage whose mean-square value is:

\[ V_{n1}^2 = \langle V_{n1}^2 + V_{n2}^2 \rangle = V_{n1}^2 + 2V_{n1}V_{n2} + V_{n2}^2 \ldots \] (1)

Because \( V_{n1} \) and \( V_{n2} \) are statistically independent (uncorrelated), the mean value of the product term in equation 1 is zero. Thus:

\[ V_{n1}^2 = V_{n1}^2 + V_{n2}^2 = 4kT (R_1 + R_2) b \]

When noise sources are uncorrelated, the results show that superposition can be used to calculate the total mean-square noise voltage.

The second method for analyzing the effect of circuit noise models the noisy circuit as a noiseless circuit plus external noise sources. For a noisy two-port network with internal noise sources (figure 2a), the effect of those sources can be represented by the external noise-voltage sources \( V_{n1} \) and \( V_{n2} \), placed in series with the input and output terminals, respectively (figure 2b). Those sources must produce the same noise voltage at the circuit terminals as do the internal noise sources. The values of \( V_{n1} \) and \( V_{n2} \) are calculated as follows: Representing the noise-free two-port network in figure 2b by its Z parameters, we can write:

\[ V_1 = Z_{11} I_1 + Z_{12} I_2 + V_{n1} \] (2)

and

\[ V_2 = Z_{21} I_1 + Z_{22} I_2 + V_{n2} \] (3)

Equations 2 and 3 show that the \( V_{n1} \) and \( V_{n2} \) values can be determined from open-circuit measurements in the noisy two-port network. It follows from these equations that when the input and output terminals are open \( (I_1 = I_2 = 0) \),

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Figure 1. The total noise voltage produced by two resistors in series is modeled as shown on the right.
In other words, $V_{n1}$ and $V_{n2}$ equal the corresponding open-circuit voltages.

In an alternate representation of the noisy two-port network (figure 3), the external sources are the current-noise sources $I_{n1}$ and $I_{n2}$. Representing the noise-free two-port network, we can write:

$$V_1 = Z_{11}(1 - I_{n1}) + Z_{12}I_{n2} + V_n$$
$$= Z_{11}I_{n1} + Z_{12}I_{n2} + (V_n - Z_{11}I_{n1})$$
\hspace{1cm} (4)

and

$$V_2 = Z_{21}(1 - I_{n1}) + Z_{22}I_{n2}$$
$$= Z_{21}I_{n1} + Z_{22}I_{n2} - Z_{11}I_{n1}$$
\hspace{1cm} (5)

Comparing (2) and (3) with (4) and (5), it follows that:

$$V_{n1} = V_n - Z_{11}I_{n1}$$
\hspace{1cm} (6)

$$V_{n2} = V_n - Z_{21}I_{n1}$$
\hspace{1cm} (7)

Hence, solving (6) and (7) for $V_n$ and $I_{n1}$ gives:

$$V_n = V_{n1} - \left( \frac{Z_{11}}{Z_{21}} \right) V_{n2}$$
\hspace{1cm} and

$$I_{n1} = -\left( \frac{V_{n2}}{Z_{21}} \right)$$
\hspace{1cm} (8)

An alternate method for determining $V_n$ and $I_{n1}$ relates them to the noise sources $V_{n1}$ and $V_{n2}$ in figure 2b. Using $Z$ parameters to represent the noise-free two-port network in figure 4, it can be written:

$$V_1 = Z_{11}(1 - I_{n1}) + Z_{12}I_{n2} + V_n$$
$$= Z_{11}I_{n1} + Z_{12}I_{n2} + (V_n - Z_{11}I_{n1})$$
\hspace{1cm} (4)

and

$$V_2 = Z_{21}(1 - I_{n1}) + Z_{22}I_{n2}$$
$$= Z_{21}I_{n1} + Z_{22}I_{n2} - Z_{11}I_{n1}$$
\hspace{1cm} (5)

Using $Z$ parameters to represent the noise-free two-port network, we can write:

$$V_1 = AV_2 + B(-I_{n2}) + V_n$$
\hspace{1cm} and

$$I_1 = CV_2 + D(-I_{n2}) + I_{n1}$$
\hspace{1cm} (9)

The previous equations show there is no simple way to evaluate $V_n$ and $I_{n1}$ in figure 4 using open- and short-circuit measurements. From a practical point of view, those values ($V_n$ and $I_{n1}$) can be expressed in terms of the noise voltages $V_n$ and $V_{n2}$ in figure 2b which require only open-circuit measurements. The relationship between noise sources $V_n$ and $I_{n1}$ in figure 4 and noise source voltages $V_{n1}$ and $V_{n2}$ in figure 2b is derived from the following.

Because $I_{n1} = -I_{n1} + V_n Y_n$, it follows that the mean square of $I_{n1}$ is given by:

$$\bar{I}_{n1}^2 = \bar{I}_{n1}^2 + V_n Y_n \bar{V}_n$$
\hspace{1cm} (11)

And, because noise from the source and noise from the two-port network are uncorrelated, we have:

$$\bar{I}_{n1}^2 = \bar{I}_{n1}^2 + (I_{n1} + V_n Y_n)^2$$
\hspace{1cm} (12)

Substituting (12) into (10) gives:

$$\bar{I}_{n1}^2 = \bar{I}_{n1}^2 + \left( \frac{V_n Y_n \bar{V}_n}{\bar{I}_{n1}^2} \right)^2$$
\hspace{1cm} (13)

Subtracting (12) into (10) gives:

$$\bar{I}_{n1}^2 = \bar{I}_{n1}^2 + \left( \frac{V_n Y_n \bar{V}_n}{\bar{I}_{n1}^2} \right)^2$$
\hspace{1cm} (13)

There is some correlation between the external sources $V_n$ and $I_{n1}$. Hence, $I_{n1}$ can be written as the sum of two terms, one uncorrelated to $V_n$ ($I_{unc}$) and one correlated to $V_n$ ($I_{nc}$). Thus:

$$I_{n1} = I_{unc} + I_{nc}$$
\hspace{1cm} (14)

Furthermore, defining the relation between $I_{nc}$ and $V_n$ in terms of a correlation admittance, $Y_c$ gives:

$$I_{nc} = Y_c V_n$$
\hspace{1cm} (15)

However, $Y_c$ is not an actual admittance in the circuit. It is defined by (15) and calculated as follows:

$$I_{nc} = I_{unc} + Y_c V_n$$
\hspace{1cm} (16)

Multiplying (16) by $V_n^*$, taking the
The noise factor can be minimized by properly selecting \( Y_s \). From (21), \( F \) is decreased by selecting \( B_s = -B_c \) (22)

Hence, from (21):

\[
F_{\text{min}} = F | Y_s = 1 + \left( \frac{G_u}{G_{\text{opt}}} + \frac{R_n}{G_{\text{opt}}} \right) (G_{\text{opt}} + G_c) \] (26)

Solving (24) for \( G_u/G_{\text{opt}} \) and substituting into (28), the expression for \( F \) can be simplified to:

\[
F = F_{\text{min}} + 4R_n \left| \frac{G_u + G_c}{G_{\text{opt}}} \right|^2 \] (31)

When the noise figure \( F \) in (31) is expressed as a function of a circle, it can be used with a Smith chart for optimum noise-figure matching in specific applications as shown in the following set of equations.
For LNA input matching, a noise circle is positioned on the Smith chart's center and radius.

Center:

\[
28 \frac{F - F_{\text{min}}}{4r_n} = \frac{\left| \Gamma_s - \Gamma_{\text{opt}} \right|^2}{\left| \Gamma - \Gamma_{\text{opt}} \right|^2}
\]

Radius:

\[
N = \frac{\left| \Gamma_s - \Gamma_{\text{opt}} \right|^2}{\left| \Gamma - \Gamma_{\text{opt}} \right|^2}
\]

Designing for Optimum Noise Figure

For any two-port network, the noise figure gives a measure of the amount of noise added to a signal transmitted through the network. For any practical circuit, the signal-to-noise ratio at its output will be worse (smaller) than at its input. In most circuit designs, however, you can minimize the noise contribution of each two-port network through a judicious choice of operating point and source resistance.

The preceding section demonstrates that for each LNA (indeed, for any two-port network) there exists an optimum noise figure. LNA manufacturers often specify an optimum source resistance on the data sheet.

To design an amplifier for minimum noise figure, determine (experimentally or from the data sheet) the source resistance and bias point that produce the minimum noise figure for that device. Next, force the actual source impedance to "look like" that optimum value. All stability considerations still apply, of course. If the calculated Rollet stability factor \( K \) is less than 1 (\( K \) is defined in the literature as a figure of merit for LNA stability), then the source and load-reflection coefficients must be carefully chosen. For an accurate graphical depiction of the unstable regions, it is best in that case to draw stability circles.

After providing the LNA with optimum source impedance, the next step is to determine the optimum load-reflection coefficient \( \Gamma_L \) needed to properly terminate the LNA's output:

\[
\Gamma_{\text{opt}} = \frac{1}{1 + N}
\]

with

\[
O_N = \frac{\frac{\Gamma_{\text{opt}}}{\left(1 + N\right)}}{1 + \frac{\Gamma_{\text{opt}}}{\left(1 + N\right)}}
\]

\[
N = \frac{F - F_{\text{min}}}{4r_n} \left[\frac{\Gamma_{\text{opt}}}{1 + \Gamma_{\text{opt}}}\right]^2
\]

\[
R_{\text{opt}} = \frac{1}{\left(1 + N\right)} \sqrt{N^2 + N \left[\frac{\Gamma_{\text{opt}}}{\left(1 + \Gamma_{\text{opt}}\right)}\right]^2}
\]

Applications

For practical examples to support the theory of optimum noise matching for LNAs, we examine an LNA (figure 6) with high third-order adjustable intercept point (IP3). This particular design is for personal communications system (PCS) phone applications with gain selected by logic control (14.5 dB in high-gain mode and 0.8 dB in low-gain mode), the amplifier exhibits an optimum noise figure of 1.9 dB (depending on the value of bias resistor, \( R_{\text{bias}} \)).

The figure 6 application employs an LNA operating at a PCS receiver frequency of 1960 MHz and noise figure of 2 dB. It must operate between 50 \( \Omega \) terminations. For this particular device, the optimum \( R_{\text{bias}} \) for minimum noise figure is 715 \( \Omega \). The optimum source-reflection coefficient \( \Gamma_{\text{opt}} \) for minimum noise figure in a 1960 MHz application is:

\[
\Gamma_{\text{opt}} = \frac{1 - S_{21} \Gamma_5 S_{12}}{S_{22} - S_{21} \Gamma_5 S_{12}}
\]

where \( \Gamma_s \) is the source-reflection coefficient necessary for minimum noise figure. (The asterisk in the above equation indicates the conjugate of the complex quantity \( \Gamma_s \).)

Figure 6. Typical operating circuit for the referenced LNA
A source impedance with noise-equivalent resistance $R_N = 43.2336 \Omega$ yields the minimum noise figure.

This particular LNA operating at 1960 MHz has the following S parameters (expressed as magnitude/angle):

- $S_{11} = 0.588/–118.67^\circ$
- $S_{21} = 4.12/149.05^\circ$
- $S_{12} = 0.03/–167.86^\circ$
- $S_{22} = 0.275/–66.353^\circ$

The calculated stability factor ($K = 2.684$) indicates unconditional stability, so we can proceed with the design.

For convenience, we choose a source-reflection coefficient of $\Gamma_S = 0.3/150^\circ$ on the 2 dB constant-noise circle. The normalized 50Ω source resistance is transformed to $\Gamma_L$ using three components: The arc $\Gamma_{SA}$ (clockwise in the impedance chart) gives the value of series inductance $L_1$. Arc $B_O$ (clockwise in the admittance chart) gives the value of shunt capacitor $C_1$. The value of arc $F_{SA}$ measured on the plot is 0.3 units, so $Z = 50(0.3) = 15\Omega$. Thus, $L_1 = 15/(2\pi f) = 15/(2\pi)(1.96 \times 10^9) = 1.218 \text{ nH}$, rounded to 1.2 nH.

Value of the arc $B_O$ measured on the plot is 0.9 units, so $1/Y = Z = 50/0.9 = 55.55 \Omega$. Thus, $C_2 = 1/(55.55(\omega)) = 1/(55.55)(2\pi f)(1.96 \times 10^9) = 1.46 \text{ pF}$, rounded to 1.5 pF.

Capacitor $C_1$ is just a high-valued DC isolation capacitor, and does not interfere with the input matching. The chosen $\Gamma_S$ provides the load-reflection coefficient needed to properly terminate the LNA:

$$\Gamma_L = \frac{S_{21}S_{12}S_{22}}{1 - S_{11}S_{22}} = 0.236/70.5^\circ$$

This value and the normalized load-resistance value are plotted in Figure 8, which also shows a possible method for transforming the 50Ω load into $\Gamma_L$. For this example, note that a single series capacitor provides the necessary impedance transformation.

The arc $\Gamma_{O_3}$ (counterclockwise in the impedance chart) gives the value for series capacitor $C_3$. The value of arc $F_{O_3}$ measured on the plot is 0.45 units, so $Z = 50(0.45) = 22.5\Omega$. Thus, $C_3 = 1/(22.5(\omega)) = 1/(22.5)(2\pi f)(1.96 \times 10^9) = 3.608 \text{ pF}$, rounded to 3.6 pF.

References


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