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"Solitons in Electrical Networks"

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Keywords: solitons, electrical solitons, nonlinear transmission lines (NLTLs), electrical soliton oscillators, pulse generation.

Solitons are a unique class of pulse-shaped waves that propagate in nonlinear dispersive media. In the course of propagation, they maintain spatial confinement of wave energy within the pulse shape without dispersion and exhibit remarkable nonlinear dynamics. Solitons are found throughout nature where fine balance between nonlinearity and dispersion is achieved. Dispersion alone would flatten out a pulse-shape wave during its course of propagation as different Fourier components of the wave would travel at different speeds. Nonlinearity, however, can counteract dispersion by making the pulse steepened and even topple over and break, as would be observed in the water waves approaching seashore. The first reported soliton (Russell 1834) was a mono-pulse of water wave in a narrow canal where the shallow water possessed both nonlinearity and dispersion. Other examples of solitons include vibrations in nonlinear spring-mass lattices, acoustic waves in plasmas, optical pulses in fiber optic cables, and fluxons in superconducting Josephson junctions.

Optical solitons in fiber optic cables are of particular technological significance. Because of their ability to propagate over long distance without dispersion of energy, the light-wave solitons can carry a large amount of digital information in long-distance communication. Laboratory experiments demonstrated that optical solitons in a single optical fiber can transmit 1 terabit of data per second over 1000 kilometers.

In electronics, the nonlinear transmission line (NLTL) serves as a nonlinear dispersive medium that propagates voltage solitons. These electrical solitons on the NLTL have been actively investigated since 1960's, particularly in the microwave domain. Unlike optical solitons, electrical solitons are not suitable for long-distance communication because of the high resistive loss present in the NLTL. Instead, they are primarily utilized for sharp pulse generation, which is of considerable interest in modern electronics. In ultra-fast time-domain metrology, the short duration of a pulse directly translates to high temporal resolution, and hence, narrow electrical pulses can be used to sample rapidly varying signals or as probe signals for high-precision time-domain reflectometry (TDR). In addition to high-resolution metrology, periodic sharp pulse trains can be utilized for impulse radar ranging or in ultra-wideband (UWB) communication systems.

The Nonlinear Transmission Line

The NLTL can be constructed from a linear transmission line (two conductors running in parallel) by periodically loading it with voltage-dependent capacitors (varactors) such as reversed-biased *pn* junction diodes, Schottky diodes, or metal-oxide-semiconductor (MOS) capacitors (Fig 1a). Alternatively, the NLTL can be obtained by forming an artificial *LC* transmission line with varactors and discrete inductors (Fig 1b). The NLTL is a nonlinear dispersive medium. The nonlinearity originates from the varactors, whose capacitance changes with the applied voltage. The dispersion arises from the structural periodicity, i.e., periodic lumped loading of varactors. A common feature of any periodic structure like the NLTL is the existence of a cutoff frequency, beyond which no Fourier component can propagate, and below which different Fourier components travel at different speeds (i.e. dispersion).

The NLTL was first introduced in 1960 by Rolf Landauer at IBM for parametric amplification. It was later shown by several scientists that the NLTL represented a discrete version of Korteweg & de Vries (KdV) equation, the governing nonlinear differential equation of shallow water wave solitons, mechanical lattice solitons, and plasma solitons, and, as such, the properties of voltage solitons on the NLTL are common among all the KdV solitons. In 1970's and 80's, the NLTL was at first used as a convenient experimental platform for studying KdV soliton dynamics since voltage signals are easily probed with oscilloscopes. Researchers soon learned, however, that this structure could be used for engineering applications due to the narrow soliton pulses it produced.

Electrical Solitons on NLTL

As mentioned shortly before, the electrical solitons are described by KdV equation, which was derived by Korteweg and de Vries in 1895 to model the dynamics of shallow water wave solitons. The general traveling wave solution on the NLTL obtained by solving the KdV equation is a periodic train of pulses known as a cnoidal wave (Fig 1c). For a given NLTL, an infinite number of cnoidal waves are possible via different interdependent combinations of the amplitude V_0, pulse spacing Λ, and pulse width W. Initial or boundary conditions will determine a specific cnoidal wave that can propagate on the NLTL. By letting the pulse spacing go to infinity, one obtains the mono-pulse soliton solution, which has a profile of sech^2. The sech^2 waveform was first given by Boussinesq (1871) and Lord Rayleigh (1876) to account for Russell's shallow water wave soliton. For high enough amplitude that results in considerable nonlinearity, the cnoidal wave can be well approximated as a periodic train of sech^2 pulses. For simplicity, we will refer to cnoidal waves as a train of solitons, although, strictly speaking, "soliton" refers only to the mono-pulse solution.

Electrical solitons or general KdV solitons exhibit fascinating properties. To begin with, the wave velocity and pulse width are closely related to the pulse amplitude. The higher the amplitude, the more prominent the nonlinear effect, and hence the steeper and narrower the soliton pulses. Since the capacitance of the varactors decreases as the applied voltages increase, solitons with higher amplitudes also experience smaller capacitance, which directly translates to smaller *LC* delays and a greater wave velocity. Due to this amplitude-dependent speed, if a taller soliton is placed behind a shorter one (Fig. 2a, top), the taller one will catch up with the shorter one and move ahead of it after a collision. During the collision (Fig. 2a, middle), the two solitons interact very strongly and experience a significant amplitude modulation (nonlinear collision). After the collision (Fig. 2a, bottom), the two solitons return to their original shapes, but have acquired a permanent time (phase) shift shown by the difference in d_1 and d_2 in Fig. 2a. It is this particle-like character (strong interaction while retaining their identities) that led Zabusky and Kruskal (1965) to coin the name “soliton” in the spirit of calling individual particles with names ending with –on, e.g., photon, electron, neutron, and person.

Non-soliton waves can also propagate on the NLTL, but only by changing their shape to form into a soliton or multiple solitons with a dispersive tail. The evolution of non-soliton waves into solitons can be analyzed by using the Inverse Scattering Method, a powerful mathematical tool developed by Gardner, Greene, Kruskal, and Miura at Princeton in 1967. Phenomenologically speaking, a non-soliton input close to soliton profile will be sharpened into the soliton (Fig. 2b, top), while one significantly different from soliton shape will break up into multiple solitons of different amplitudes (Fig. 2b, bottom). In either case, the input pulse width gets compressed traveling down the line. Once a soliton or solitons are formed, they propagate without further sharpening or breakup. The past 40 years have seen this transient electrical soliton-forming process occupying an especially significant place in modern electronics for sharp pulse generation.

NLTL as a Two-Port System for Pulse Compression

The generation of picosecond pulses from electrical solitons propagating on an NLTL was proposed by Riley at Stanford in 1961. The active studies of narrow pulse generation on the NLTL during the past 40 years have culminated in the state-of-the-art monolithic NLTL that can achieve a pulse width down to the low picosecond range (< 10 ps). In these fascinating developments, the NLTL has been used as a two-port system (input and output ports), which utilizes the transient soliton-forming process to compress an external high-frequency input signal into narrow soliton output pulses. The advantage of using the NLTL for sharp pulse generation is that the pulse rise time is no longer limited by the speed of the transistors in the active circuitry, but by the *RC* time constant of the varactors, which are usually much faster than the transistors. Especially Schottky diode varactors can be as fast as a couple of terahertz.

Electrical pulse compression by a factor of 2 can be achieved with a homogeneous (constant cutoff frequency) NLTL. For higher order pulse compression, NLTLs with higher cutoff frequencies (smaller *LC* time constants) must be used to allow the propagation of the high frequency Fourier components of the compressed pulse. As briefly mentioned in the previous section, however, this would inevitably result in the pulse breaking into multiple soliton pulses due to the significant difference between the shapes of the input pulse and the sharp solitons supported by the NLTL. Tan, Su, and Anklam (1988) at Hewlett-Packard introduced a tapered nonlinear transmission line consisting several cascaded homogeneous sections of NLTL with increasing cutoff frequency (i.e. decreasing *LC* time constants & weaker dispersion). A pulse passing through the cascaded sections will be successively compressed, and the *LC* time constants are chosen so that only a single soliton will arise during the course of compression. Pulse compression by a factor of 7 has been demonstrated with such tapered NLTLs. Rodwell (UC Santa Barbara) and van der Weide (Max-Planck-Institute) adapted Tan’s technique and integrated it on a monolithic substrate with state-of-the-art GaAs technologies to produce the fastest electrical pulses ever generated from an all-electrical system. Rodwell’s student, Case, achieved a pulse width of 5 ps while van der Weide demonstrated an edge rise time of 480 fs.

These ultra-fast NLTLs have been used in numerous applications, in particular ultra-fast metrology, where a sampling bandwidth of over 700 GHz has been achieved.

The disadvantage of this two-port topology is that it requires an external high-frequency input. In addition, due to the absence of feedback, the output waveform is sensitive to the quality and shape of the input signal.

Electrical Soliton Oscillator

One meaningful extension of the two-port NLTL works would be to construct a one-port (ouput port only) self-sustained electrical soliton oscillator by properly combining the NLTL with an amplifier (positive active feedback), as shown in Fig. 3. Such an oscillator would self-start by growing from ambient noise to produce a periodic train of soliton pulses in steady state, and hence would make a self-contained soliton generator not requiring an external high-frequency input.

In 1990's, Ballantyne *et al* demonstrated an NLTL-based soliton oscillator with a linear amplifier. Its oscillations, however, were not always reproducible, lacking robustness and controllability. The difficulty arises, as the NLTL's soliton dynamics do not easily lend themselves to standard amplification techniques. In the case of a linear amplifier, for example, any small perturbation from noise, non-ideal termination, and the dispersive tail is non-discriminatively amplified and continues to grow into a soliton. These solitons with various amplitudes would circulate in the oscillatory loop of Fig. 3 at different speeds and continually collide with one another, resulting in amplitude and pulse repetition rate variations in oscillations, often tending towards chaotic states.

The first robust self-sustained electrical soliton oscillator was recently developed by Ricketts, Li, and Ham (2005) at Harvard. The oscillator employs a special nonlinear amplifier with adaptive bias control to "tame" the unruly dynamics of solitons on the NLTL. Initially, the amplifier is biased in the gain region, which allows startup from ambient noise. As the oscillation grows and forms into pulses, the *dc* component of the output waveform steadily increases. The amplifier uses this increase in the *dc* component to lower its bias point such that the small perturbations are attenuated while the large signal pulses still receive enough gain without significant distortion. This threshold-dependent gain-attenuation mechanism is a technique widely employed in modelocked lasers in optics, where it is known as *saturable absorption,* but was originally introduced in electronics during the vacuum-tube era by Cutler (1955) for his linear pulse oscillator. The proof-of-concept prototype implemented in CMOS integrated circuit produces a train of solitons with 1.14 GHz repetition rate and a pulse width of 293 ps. Although the pulse widths achieved are not record numbers as compared to currently available electrical pulse generation techniques, including the two-port pulse compression with NLTL, the soliton oscillator, especially its NLTL, can be quickly scaled down and optimized to generate much narrower pulses.

In summary, electrical solitons on the NLTLs have been actively investigated in the past 40 years for sharp electrical pulse generation applications. Two major topologies have been developed. The record-holding ultra-fast GaAs NLTL used as a two-port passive network can produce pulses as narrow as a few picoseconds. The recently developed electrical soliton oscillator provides a new approach to robust self-sustained all-electrical soliton pulse generation.

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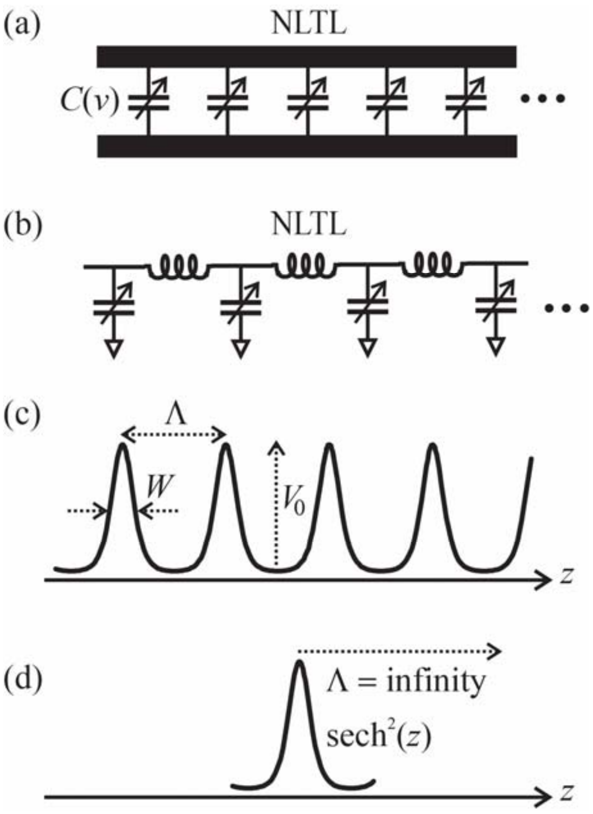

Figure 1

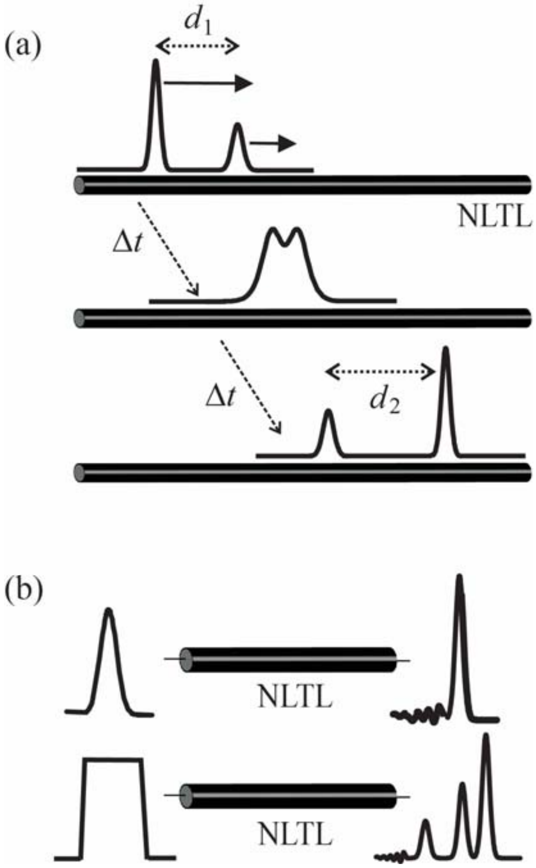

Figure 2

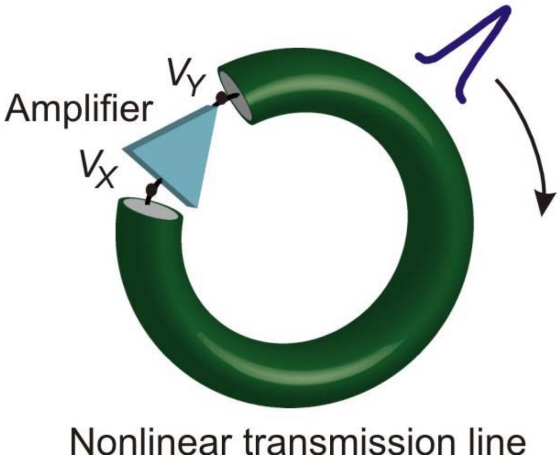

Figure 3